# Re-Examining the Profitability of Technical Analysis with White's Reality Check and Hansen's SPA Test 

Po-Hsuan Hsu<br>Department of Finance and Economics<br>Graduate School of Business<br>Columbia University

## Chung-Ming Kuan

Institute of Economics
Academia Sinica

This version: March 8, 2005

Author for correspondence: Chung-Ming Kuan, Institute of Economics, Academia Sinica, Taipei 115, Taiwan; ckuan@econ.sinica.edu.tw; phone: +886 2 2782-2791 ext. 646, fax: +886 2 2782-2019.
$\dagger$ We are grateful to two anonymous referees and Jeff Russell, the guest editor, whose comments lead to a much improved version of the paper. An early version of this paper, entitled "Re-Examining the Profitability of Technical Analysis with White's Reality Check," was presented at the 2003 Taipei conference on "Analysis of High-Frequency Financial Data and Market Microstructure." We thank A. Ang, M. Chernov, P. R. Hansen, G. Huberman, W. Jiang, M. Johannes, C. Jones, B. Lehmann, C. J. Neely, M. J. Ready, A. Timmermann, and the participants of the 2003 Taipei conference and the finance doctorate seminar at Columia GSB, for their valuable comments and suggestions. We also thank S.-J. Lin, J. Shyu and H.-C. Yu for their comments on the preliminary version of this paper. Assistance from H.-H. Lee, Y.-C. Tai and C.-H. Wang on computing is much appreciated. All remaining errors are ours.


#### Abstract

In this paper, we re-examine the profitability of technical analysis using White's Reality Check and Hansen's SPA test that correct the data snooping bias. Comparing to previous studies, we study a more complete "universe" of trading techniques, including not only simple rules but also investor's strategies, and we test the profitability of these rules and strategies with four main indices. It is found that significantly profitable simple rules and investor's strategies do exist in the data from relatively "young" markets (NASDAQ Composite and Russell 2000) but not in the data from relatively "mature" markets (DJIA and S\&P 500). Moreover, after taking transaction costs into account, we find that the best rules for NASDAQ Composite and Russell 2000 outperform the buy-and-hold strategy in most in- and out-of-sample periods. It is also found that investor's strategies are able to improve on the profits of simple rules and may even generate significant profits from unprofitable simple rules.


Keywords: data snooping, investor's strategies, SPA test, stationary bootstrap, technical analysis, trading rules, White's Reality Check.

## 1 Introduction

Technical analysis has been widely applied by practitioners to analyze financial data and make trading decisions for decades. This method relies on mechanical trading rules and strategies to generate buy and sell signals. Thus, whether these trading techniques indeed result in significant profit has been a long-debated issue since Fama and Blume (1966). Recent empirical studies, however, find more and more supporting evidences for the profitability of techncial analysis, including, among others, Sweeney (1986, 1988), Brock, Lakonishok, and LeBaron (1992), Blume, Easley, and O'Hara (1994), Chan, Jegadeesh, and Lakonishok (1996, 1999), Gencay (1998, 1999), Neely, Weller, and Dittmar (1997), Brown, Goetzmann, and Kumar (1998), Rouwenhorst (1998), Allen and Karjalainen (1999), Chang and Osler (1999), Neely and Weller (1999), Chan, Hameed, and Tong (2000), and Lo, Mamaysky, and Wang (2000). These results suggest that technical analysis is popular because it can "beat the market."

Lo and MacKinlay (1990) and Brock, Lakonishok, and LeBaron (1992), on the other hand, raised a concern about the data snooping bias in many empirical studies. Such bias is mainly a consequence of data reuse. In the context of evaluating technical analysis, it is conceivable that, by repeatedly examining different trading rules using the same data set, some rules would appear to be profitable, yet such profitability may simply be due to luck. This concern is shared by academic and market professionals; see, e.g., Allen and Karjalainen (1999), LeBaron and Vaitilingam (1999), and Ready (2002). To avoid spurious inferences resulted from data snooping, White (2000) proposed a formal test, now also known as White's Reality Check, on whether there exists a superior model (rule) in a "universe" of models (rules). Sullivan, Timmermann, and White (1999), henceforth STW, and White (2000) applied this test and found that there exists no significantly profitable trading rule for Dow Jones Industrial Average (DJIA) index, S\&P 500 index, and S\&P 500 futures.

There are, however, some limitations of STW's analysis. To quantify the effect of data snooping, White's Reality Check requires constructing a "universe" of the trading rules considered by previous researchers and practitioners. To this end, STW collected 7, 846 trading rules, drawn from 5 commonly used classes of rules in financial markets. Although 7,846 is a large number, this collection of rules is far from complete because: (1) it does not include several well known classes of trading rules, such as momentum strategies and head-and-shoulders, and (2) it includes only simple rules but not investor's
strategies. In practice, an investor need not stick to only one simple rule but may employ a complex trading strategy that utilizes the information from many simple rules. These rules and strategies can enlarge the "effective span" of the trading rules studied in STW and hence may affect the result of Reality Check. Moreover, STW analyzed only the samples of more "mature" markets, such as DJIA and S\&P 500. Since the last decade, small-cap and technology stocks have played more active roles in contemporary financial markets. It is therefore also interesting to find out whether STW's claim remains valid in the samples of relatively "young" markets.

In addition to the aforementioned limitations, White's Reality Check also suffers from some drawbacks. Recall that the null hypothesis of Reality Check is composite, yet its distribution is obtained based on the "least favorable configuration," i.e., a point in the null that is least favorable to the alternative. Hansen (2004) showed that the performance of Reality Check can be affected by poor (or irrelevant) trading rules that deviate largely from the least favorable configuration. In fact, its power can be driven to zero by adding sufficiently many poor rules to the "universe" being tested. Furthermore, Reality Check has power disadvantage because it is not based on standardized statistics. To circumvent these drawbacks, Hansen (2004) proposed a more powerful test for superior predictive ability (SPA). Basing on this SPA test, Hansen, Lunde, and Nason (2004) found signficant calendar effects, in contrast with the result based on Reality Check (Sullivan, Timmermann, and White, 2001).

In this paper, we extend the analysis of STW and White (2000) along the following lines. First, we consider an expanded "universe" of 39,832 simple trading rules, "contrarian" rules, and investor's strategies. Second, both White's Reality Check and Hansen's SPA test are employed to evaluate these rules and strategies. Hansen's test is particularly relevant here because we consider a much large "universe" of trading rules. Third, our study covers the indices of both "mature" and "young" markets: DJIA, S\&P 500, NASDAQ Composite, and Russell 2000. Fourth, we consider transaction costs in evaluating the performance of trading rules. It is found that profitable trading rules and investor's strategies do exist with statistical significance for NASDAQ Composite and Russell 2000. Yet the claims of STW and White (2000) still stand for DJIA and S\&P 500 even when the more powerful SPA test is used. It is also interesting to observe that some investor's strategies constructed from unprofitable simple rules can generate significant profits. Further examination shows that, when transaction costs are taken into account, the best rules for NASDAQ Composite and Russell 2000 outperform the
buy-and-hold strategy in most in- and out-of-sample periods. Our results are thus in line with the claim that market efficiency is related to the degree of market maturity.

This paper is organized as follows. White's Reality Check and Hansen's SPA test are reviewed in Section 2. The trading rules and investor's strategies included in our expanded "universe" are described in Section 3. Section 4 presents the empirical results. Section 5 concludes the paper. The parameter values of the trading rules and strategies are given in the Appendix.

## 2 Reality Check and SPA Test

The data snooping bias may arise when previous test results based on the same data set are ignored. Lo and MacKinlay (1990) showed that even slight prior information has a dramatic impact on the resulting statistical inferences. There are basically two approaches to tackling the data snooping bias. The first approach avoids re-using the same data set by testing a model with a different but comparable data set (Lakonishok, Shleifer, and Vishny, 1994; Chan, Karceski, and Lakonishok, 1998) or by validating the test using subsamples of a large data set (Brock, Lakonishok, and LeBaron, 1992; Rouwenhorst, 1998, 1999; Gencay, 1998) In practice, comparable data sets may not be available; yet sample splitting is somewhat arbitrary and hence lacks desired objectivity. A more formal approach is to consider all possible models and properly control the test size (type I error). This may be done using the Bonferroni inequality. This method is, however, not appropriate when the number of hypotheses (models) is large, as in the case of testing the profitability of technical analysis, e.g, Lakonishok and Smidt (1988). Following the second approach, White (2000) and Hansen (2004) proposed tests that are applicable when there are many models.

Let $\varphi_{k}(k=1, \ldots, M)$ denote a performance measure of the $k$-th model (rule) relative to the benchmark model (rule). The null hypothesis is that there does not exist a superior model (rule) in the collection of $M$ models (rules):

$$
\begin{equation*}
H_{0}: \max _{k=1, \ldots, M} \varphi_{k} \leq 0 \tag{1}
\end{equation*}
$$

Rejecting (1) implies that there exists at least one model (rule) that outperforms the benchmark. Testing this hypothesis is cumbersome when all models (rules) are evaluated using the same data set and also when $M$ is large. Setting the rule of no position (zero return) at all time as the benchmark, we have $\varphi_{k}=\mathbb{E}\left(f_{k}\right)$, where $f_{k}$ is the return of
the $k$-th trading rule. It is then natural to base a test of (1) on the maximum of the normalized sample average of $f_{k, t}$ :

$$
\begin{equation*}
\bar{V}_{n}=\max _{k=1, \ldots, M} \sqrt{n} \bar{f}_{k}, \tag{2}
\end{equation*}
$$

where $\bar{f}_{k}=\sum_{t=1}^{n} f_{k, t} / n$ with $f_{k, t}$ the $t$-th observation of $f_{k}$.
White (2000) suggested using the stationary bootstrap method of Politis and Romano (1994) to compute the $p$-values of $\bar{V}_{n}$. Let $f_{k}^{*}(b)$ denote the $b$-th bootstrapped sample of $f_{k}$ and $\bar{f}_{k}^{*}(b)=\sum_{t=1}^{n} f_{k, t}^{*}(b) / n$ its sample average. We then obtain the empirical distribution of $\bar{V}_{n}^{*}$ with the realizations:

$$
\begin{equation*}
\bar{V}_{n}^{*}(b)=\max _{k=1, \ldots, M} \sqrt{n}\left(\bar{f}_{k}^{*}(b)-\bar{f}_{k}\right), \quad b=1, \ldots, B . \tag{3}
\end{equation*}
$$

The Reality Check $p$-value is obtained by comparing $\bar{V}_{n}$ with the quantiles of the empirical distribution of $\bar{V}_{n}^{*}$. The null hypothesis is rejected whenever the $p$-value is less than a given significance level.

Hansen (2004) pointed out two problems with White's Reality Check. First, the average returns $\bar{f}_{k}$ are not standardized. Second, despite that the null (1) is composite, the distribution of Reality Check is based on the "least favorable configuration" to the alternative, i.e., $\mathbb{E}\left(f_{k}\right)=0$ for all $k$. Therefore, the performance of Reality Check will be adversely affected when poor models (rules) with very negative $\mathbb{E}\left(f_{k}\right)$ are included in the test. The proposed SPA test is based on studentized returns:

$$
\begin{equation*}
\widetilde{V}_{n}=\max \left(\max _{k=1, \ldots, M} \frac{\sqrt{n} \bar{f}_{k}}{\hat{\sigma}_{k}}, 0\right), \tag{4}
\end{equation*}
$$

where $\hat{\sigma}_{k}$ is a consistent estimator of the standard deviation of $n^{1 / 2} \bar{f}_{k}$, cf. (2).
To avoid using the least favorable configuration and to reduce the influence of the rules with large negative returns, Hansen (2004) suggested a different way to bootstrap the distribution of $\widetilde{V}_{n}$. For the $k$-th rule, let $\bar{Z}_{k}^{*}(b)$ denote the sample average of the $b$-th bootstrapped sample of the centered returns:

$$
\begin{equation*}
Z_{k, t}^{*}(b)=f_{k, t}^{*}(b)-\bar{f}_{k} \mathbf{1}_{\left\{\bar{f}_{k} \geq-A_{k}\right\}}, \tag{5}
\end{equation*}
$$

where $A_{k}$ involves $\hat{\sigma}_{k}$ and is a function of $n$ (we suppress the subscript $n$ ); the choice of $A_{k}$ will be discussed in Section 4.2. The consistent $p$-values of $\widetilde{V}_{n}$ is determined by the empirical distribution of $\widetilde{V}_{n}^{*}$ whose realizations are

$$
\begin{equation*}
\widetilde{V}_{n}^{*}(b)=\max \left(\max _{k=1, \ldots, M} \frac{\sqrt{n} \bar{Z}_{k}^{*}(b)}{\hat{\sigma}_{k}}, 0\right), \quad b=1, \ldots, B . \tag{6}
\end{equation*}
$$

When such $p$-values are used, the test (4) is referred to as the $\mathrm{SPA}_{C}$ test.
Moreover, Hansen (2004) considered the averages $\bar{Z}_{k}^{L *}(b)$ and $\bar{Z}_{k}^{U *}(b)$ based on the following respective bootstrapped returns:

$$
\begin{aligned}
& Z_{k, t}^{L *}(b)=f_{k, t}^{*}(b)-\max \left(\bar{f}_{k}, 0\right), \\
& Z_{k, t}^{U *}(j)=f_{k, t}^{*}(b)-\bar{f}_{k},
\end{aligned}
$$

cf. (5). It can be seen that $\bar{Z}_{k}^{L *}(b) \leq \bar{Z}_{k}^{*}(b) \leq \bar{Z}_{k}^{U *}(b)$. The bootstrapped distributions of $\widetilde{V}_{n}^{L *}$ and $\widetilde{V}_{n}^{U *}$ are now computed as (6), with $\bar{Z}_{k}^{*}(b)$ replaced by $\bar{Z}_{k}^{L *}(b)$ and $\bar{Z}_{k}^{U *}(b)$, respectively. The test $\widetilde{V}_{n}$ will be referred to as the $\mathrm{SPA}_{L}$ and $\mathrm{SPA}_{U}$ tests if its $p$-values are determined by the distributions of $\widetilde{V}_{n}^{L *}$ and $\widetilde{V}_{n}^{U *}$, respectively. Although these $p$-values are inconsistent, they serve as the lower and upper bounds for the consistent $p$-values.

## 3 An Expanded Universe of Trading Rules and Strategies

A crucial step in White's Reality Check is to construct a "universe" of trading rules and strategies for evaluation. In this paper we expand the universe of STW to a collection of 39,832 rules and strategies, including 12 classes of 18,326 simple rules, 18,326 corresponding "contrarian" rules, and 3,180 investor's strategies. Note that not only investor's strategies but also contrarian rules have not been considered in the related literature. Table 1 summarizes the classes of rules and strategies considered in this study and indicates where to find their parameter values.

### 3.1 Simple Trading Rules

The first 5 classes of simple rules in Table 1 were used by STW to form their universe; see STW for details. The other 7 classes of simple rules are also well known among market professionals and will be discussed below. Note that, among these 7 classes of rules, Momentum strategies have been widely analyzed in the literature; see e.g., LeBaron (1992), Chan, Jegadeesh, and Lakonishok (1996, 1999), Rouwenhorst (1998, 1999), Chan, Hameed, and Tong (2000), and Goetzmann and Massa (2002). All other rules were studied by Lo, Mamaysky, and Wang (2000); Chang and Osler (1999) focused on the head-and-shoulders rules.

A momentum strategy is determined by an "oscillator" constructed from a momentum measure. The momentum measure used in this study is the rate of change (ROC).

Table 1: The expanded universe of trading rules and strategies.

| Simple trading rules | $\mathbf{1 8 , 3 2 6}$ |  |
| :---: | ---: | :--- |
| Filter Rules (FR) | 497 | STW |
| Moving Averages (MA) | 2,049 | STW |
| Support and Resistance (SR) | 1,220 | STW |
| Channel Break-Outs (CB) | 2,040 | STW |
| On Balance Volume Averages (OBV) | 2,040 | STW |
| Momentum Strategies in Price (MSP) | 1,760 | A.1 |
| Momentum Strategies in Volume (MSV) | 1,760 | A.1 |
| Head and Shoulders (HS) | 1,200 | A.2 |
| Triangle (TA) | 720 | A.3 |
| Rectangle (RA) | 2,160 | A.4 |
| Double Tops and Bottoms (DTB) | 2,160 | A.5 |
| Broadening Tops and Bottoms (BTB) | 720 | A.3 |
| Contrarian trading rules | $\mathbf{1 8 , 3 2 6}$ |  |
| Investor's strategies | $\mathbf{3 , 1 8 0}$ |  |
| Learning Strategies (LS) | 1,404 | A.6 |
| Vote Strategies (VS) | 888 | A.7 |
| Position Changeable Strategies (PCS) | 888 | A.7 |
| Total | $\mathbf{3 9 , 8 3 2}$ |  |

Notes: The second column shows the number of rules in that class; the third column indicates where to find the parameter values for these rules. The parameter values for contrarian rules are the same as their corresponding simple rules.

Specifically, the $m$-day ROC at time $t$ is $\left(q_{t}-q_{t-m}\right) / q_{t-m}$, where $q_{t}$ may be the closing price (leading to a momentum strategy in price, MSP) or the closing volume (leading to a momentum strategy in volume, MSV). Pring $(1991,1993)$ recommended three oscillators: simple, moving average, and cross-over moving average oscillators. The simple oscillator is just the $m$-day ROC ; the moving average oscillator is the $w$-day moving average of $m$-day ROC with $w \leq m$; the cross-over moving average oscillator is the ratio of the $w_{1}$-day moving average to the $w_{2}$-day moving average (both based on $m$-day ROC) with $w_{1}<w_{2}$. An overbought/oversold level $k$ (say $5 \%$ or $10 \%$ ) is needed to determine whether a position should be initiated. When the oscillator crosses the overbought level from below, it is a signal for initiating a long position; a signal for a short position will
be issued when the oscillator crosses the oversold level from above. We set that, once a position is initiated, the investor will hold the position for fixed holding days $f$ and then liquidate it. There are 1,760 rules in the MSP class and 1,760 rules in the MSV class.

The head-and-shoulders (HS) rules are determined by the top-and-bottom patterns of price movements. For a given sample period with five equal subperiods, each with $n$ days, an HS pattern is such that the price sequentially exhibits left shoulder (top), left trough (bottom), head (top), right trough (bottom), and right shoulder (top) in these subperiods. We require the two shoulders (troughs) being approximately equal such that their differences are no more than a differential rate $x$. To identify this pattern more easily, it is also required that the maximal price of the head subperiod must be the highest price in all subperiods. Moreover, the minimal prices in the head and shoulder subperiods must be higher than those of adjacent trough subperiods, and the maximal prices of two trough subperiods must be lower than those of the head and shoulder subperiods. Once an HS pattern is completed, future price movement is expected to decline because it is believed that the falling trend would prevail after such a struggle of price adjustment. Thus, an HS pattern serves as a signal of taking a short position. For these trading rules, we consider three liquidation methods: fixed holding days $f$, stoploss rate $r$, and fixed liquidation price (depending on the parameter $d$ ). There are 1,200 rules in this class.

The trading rules of the triangle (TA) class are also based a series of top-and-bottom patterns. To identify a triangle, we again divide a given period into five equal subperiods, each with $n$ days, orderly numbered from 1 to 5 . Let $M_{i}$ and $m_{i}$ denote, respectively, the maximum and minimum in subperiod $i$. A triangle is formed when either one of the two patterns below holds: (1) $M_{1}, M_{3}, M_{5}$ are tops such that $M_{1}>M_{3}>M_{5}$, and $m_{2}, m_{4}$ are bottoms such that $m_{2}<m_{4}$; (2) $m_{1}, m_{3}, m_{5}$ are bottoms such that $m_{1}<m_{3}<m_{5}$, and $M_{2}, M_{4}$ are tops such that $M_{2}>M_{4}$. Moreover, the minimal (maximal) closing price of a top subperiod is required to be higher than the minimal (maximal) of adjacent bottom subperiod(s). The trading rules of the rectangle (RA) class are determined similarly. A rectangle is formed when the tops $M_{1}, M_{3}, M_{5}$ (or $M_{2}, M_{4}$ ) lie near an upper horizontal line and the bottoms $m_{2}, m_{4}\left(\right.$ or $\left.m_{1}, m_{3}, m_{5}\right)$ lie near a lower horizontal line. By "near a horizontal line" we mean the difference between the tops (bottoms) are within certain bounds (e.g., $\pm 0.005, \pm 0.0075$ ). Once a triangle (rectangle) is completed, it will be a signal of a long (short) position if the future closing price exceeds the latest top (or falls below the latest bottom) by a fixed proportion $x$, known as the "trend filter." We also considered three liquidation methods for these two classes: fixed holding days $f$, stoploss
rate $r$, and day filter $d$. TA and RA contain, respectively, 720 and 2,160 rules.
The double tops and bottoms (DTB) class includes two patterns: double-top and double-bottom. Dividing a given sample period into three equal subperiods, each with $n$ days, a double-top is formed by two equal tops (maxima) in the first and last subperiods and a bottom (minimum) in the middle. Similarly, a double-bottom is formed by two equal bottoms (minima) in the first and last subperiods and a top (maximum) in the middle The tops (bottoms) are considered equal if they are within certain bounds of their average (e.g., $\pm 0.005, \pm 0.0075$ ). To identify the double-top (double-bottom) pattern more easily, we require the minimal (maximal) closing price of the second subperiod is at least $g$ percent lower (higher) than the average of two tops (bottoms). Similar to the TA and RA classes, the minimal (maxmimal) closing price of a top subperiod must be higher than the minimal (maximal) of adjacent bottom subperiod. Also, a trend filter is needed to determine future price movement. If the closing price in a following day exceeds the latest top (falls below the latest bottom) by a trend filter $x$, it is a sign of long (short) position. We again consider three liquidation methods: fixed holding days $f$, stoploss rate $r$, and day filter $d$. There are 2,160 rules in the DTB class.

The trading rules in the broadening tops and bottoms (BTB) class, similar to those in the TA class, are determined by the top-and-bottom patterns in five subperiods. The difference is that TA requires "convergence" in shape, whereas BTB corresponds to "divergence" in shape. More specifically, again let $M_{i}$ and $m_{i}$ denote, respectively, the maximum and minimum in subperiod $i$. A BTB pattern is formed if one of the two conditions below holds: (1) $M_{1}, M_{3}, M_{5}$ are tops such that $M_{1}<M_{3}<M_{5}$, and $m_{2}, m_{4}$ are bottoms satisfying $m_{2}>m_{4}$; (2) $m_{1}, m_{3}, m_{5}$ are bottoms such that $m_{1}>m_{3}>m_{5}$, and $M_{2}, M_{4}$ are tops satisfying $M_{2}<M_{4}$. There are 720 rules in this class, and their parameters are set as in the TA class.

### 3.2 Contrarian Trading Rules

"Contrarian" rules are common in trader's handbooks (e.g., LeBaron and Vaitilingam, 1999, and Siegel, 2002), but they were rarely inspected in previous empirical studies. Corresponding to each simple trading rule, a contrarian rule is such that a long signal of the simple rule triggers a short position and vice versa. Thus, the investor following a contrarian rule simply takes the position opposite to that suggested by the corresponding simple rule. Typically, technical analysts believe that the trading signals of some trading
rules are caused by price deviations far from the current state and hence signify changes in trend. The rationale of contrarian rules is that such price deviations might still be temporary so that the market will return to its original state sooner or later. In our study, there are 18,326 simple trading rules and hence 18,326 corresponding contrarian rules.

### 3.3 Investor's Strategies

For technical analysts, investor's strategies are usually more important than simple trading rules. In practice, it is hard to believe that technical investors stick to only a single rule without incorporating other available information. Pring (1991) also argued: "No single indicator can ever be expected to signal all trend reversals, and so it is essential to use a number of them together to build up a consensus" (p. 9). Thus, investor's strategies rely on the information generated from many simple rules and make trading decisions through a complex evaluation process. Despite their practical relevance, investor's strategies have not been examined in previous studies of technical analysis. In this paper, we consider three classes of investor's strategies: learning strategies (LS), vote strategies (VS), position changeable strategies (PCS), leading to a total of 3, 180 strategies.

A learning strategy allows investors to switch their positions by following the bestperformed rule within a rule class. In this study, a rule class may be a particular class of simple rules or the collection of all simple rules ( 12 classes of rules). There are three dimensions in this class: memory span $m$, review span $r$, and performance measure. The memory span specifies the period of time for evaluating the rule performance. The review span indicates how often an investor evaluates the performance and switches the trading rule accordingly. We set $r \leq m$. We consider three performance measures: (1) the sum of $m$ daily returns; (2) the average of $m$ log daily returns; (3) the average log returns of all position-held days in the past $m$ days. If there are more than two rules that generate equivalent returns, the investor is set to follow the one that performs better in the previous evaluation. There are 1,404 learning strategies.

A vote strategy is based on the "voting" result of the trading rules in a rule class. In particular, each rule has one vote based on its suggested position. In our study, we consider two types of ballot: two-choice ballot for long and short positions and threechoice ballot for long, no, and short positions. A position is initiated if that position receives a larger proportion of votes. For the rule class, we consider every class of simple
rules separately but not the one consisting of all rules. This is to avoid the voting result being dominated by the class with a large number of rules. There are another two dimensions in this class: memory span $m$ and review span $r$, as in the LS class. There are 888 vote strategies.

The position changeable strategies differ from learning and vote strategies in that they allow for non-integral positions. Typically, a trading rule or strategy issues a signal of a specific position. Edwards and Magee (1997, pp. 535-540) proposed using an "evaluation index" to determine how a position can be divided. In this study, the voting results of the VS class serve as the evaluation index. As there are two types of ballot, there are also two evaluation indices. Each index is the percentage of the winning votes, and the resulting position is exactly the percentage of the winning votes. There are 888 strategies in the PCS class; the parameter values are the same as those in the VS class.

## 4 Empirical Results

### 4.1 Data

In our empirical studies, the trading rules and strategies discussed in the preceding section are applied to four main indices: DJIA, S\&P 500, NASDAQ Composite, and Russell 2000. Our analysis is based on the daily returns computed using daily closing prices of these indices. This kind of study makes practical sense because the trading rules and strategies utilize only public information available after the market closes. Moreover, as these indices are the targets of numerous index funds, our results would be informative to those "big players."

The daily index data from 1989 through 2002 are obtained from the Commodity System Inc. The in-sample period is from 1990 through 2000 with 2779 observations; the data of 2001 (248 observations) and 2002 (252 observations) are reserved for out-ofsample evaluation. The data of 1989 (252 observations) are only used to formulate rules and strategies in 1990 that require the information from previous year. This data set extends more than a decade and may mitigate potential data snooping to some extent (Merton, 1987). Note that the volume data of DJIA are the share volumes of 30 stocks in DJIA; the volume data for NASDAQ Composite are the total share volume in NASDAQ. Because the exact share volume of S\&P 500 is not available, we use the total share volume of New York Stock Exchange (NYSE) as a proxy because S\&P 500 stocks amount more
than $3 / 4$ of the market capitalization in NYSE. For Russell 2000, since neither the exact volume data nor an appropriate proxy is available, we exclude the rules and strategies that require the information on volume. Therefore, there are only 35,776 rules and strategies for testing Russell 2000.

### 4.2 Implementing the Reality Check and SPA test

We apply White's Reality Check and Hansen's SPA test to the expanded universe based on mean returns. It must be mentioned that the rules in the HS, TA, RA, DTB and BTB classes generate much less trading signals than do the other trading rules during the sample period. The resulting mean returns therefore may not be directly comparable with the results of other rules. As such, we adopt a modified approach to computing the returns of these five classes: the investor holds double positions when there is a long signal, one position when there is no signal, and no position when a short signal is issued. ${ }^{1}$

The Reality Check statistic $\bar{V}_{n}$ is computed according to (2), where $n=2,779$ for insample evaluations. To compute the Reality Check $p$-values, 1,000 bootstrapped samples are obtained by resampling the $n \times M$ return matrix $\left\{\eta_{k, t}=y_{t} s_{k, t-1}\right\}, t=1, \ldots, n$, $k=1, \ldots, M$, where $s_{k, t-1}$ is the signal function of the $k$-th rule based on the information up to time $t-1$ such that it takes the value 1 for a long position, 0 for no position, or -1 for a short position. Each resampled return matrix is computed as follows.

1. Randomly select a row $\left(\eta_{t, 1}, \ldots, \eta_{t, M}\right)$ of the original return matrix as the first resampled row $\left(\eta_{1,1}^{*}, \ldots, \eta_{1, M}^{*}\right)$.
2. The second resampled row $\left(\eta_{2,1}^{*}, \ldots, \eta_{2, M}^{*}\right)$ is randomly selected from the original return matrix with probability $q$, or it is set to the next row of the previously resampled row, i.e., $\left(\eta_{t+1,1}, \ldots, X_{t+1, M}\right)$, with probability 1- $q$.
3. Repeat the second step to form an $n \times M$ resampled return matrix. ${ }^{2}$

From the $b$-th resampled return matrix, it is easy to compute $\bar{f}_{k}^{*}(b)$ and hence $\bar{V}_{n}^{*}(b)$ in (3). The significance of $\bar{V}_{n}$ is determined by the empirical distribution of $\bar{V}_{n}^{*}$.

[^0]To compute the SPA tests, we must find an estimator $\hat{\sigma}_{k}$ for the standard deviation of the $k$-th rule's average returns. In our study, this estimator is

$$
\hat{\sigma}_{k}^{2}=\frac{1}{n} \sum_{t=1}^{n}\left(f_{k, t}-\bar{f}_{k}\right)^{2}
$$

one may also compute $\hat{\sigma}_{k}^{2}$ based on boostrapped re-samples. ${ }^{3}$ For the $\mathrm{SPA}_{C}$ test, we also need to determine the threshold $A_{k}$ in (5). There is a wide range of valid choices for $A_{k}$; we follow Hansen (2004) and set $A_{k}=\hat{\sigma}_{k} /\left(4 n^{1 / 4}\right)$. The bootstapped distributions of $\widetilde{V}_{n}^{L *}, \widetilde{V}_{n}^{*}$ and $\widetilde{V}_{n}^{U *}$ are computed similarly to that of Reality Check discussed above.

In this study, all programs were written in Fortran and S-plus 2000. We checked our programs by comparing the returns of technical rules with those mentioned in STW and Brock, Lakonishok, and LeBaron (1992) and found the results are quite close. Similar to STW, we also found that the probability parameter $q=0.01,0.1$, and 0.5 in stationary bootstrap yield similar results. We therefore report only the results under $q=0.1$.

### 4.3 Profitable Rules and Strategies

We first examine the profitability of the trading rules for the in-sample data of 1990 through 2000. In what follows, the rule with the largest (standardized) mean return for each index will be referred to as the best (standardized) rule. Table 2 summarizes the daily returns, annual returns (daily returns times 252) and Reality Check $p$-values of the best rules. As SPA tests are based on standardized mean returns, the daily returns, annual returns and SPA $p$-values of the best standardized rules are summarized in Table 3.

We observe that for DJIA (NASDAQ Composite), the best rule and best standardized rule are the same momentum strategy in volume (2-day MA rule), yet for S\&P 500 and Russell 2000, the best rules do not coincide. For DJIA and S\&P 500, the annual returns of these best rules are high (around $15 \%$ ) but their $p$-values based on Reality Check and SPA tests are all very large. In particular, the Reality Check p-values for the returns in DJIA and S\&P 500 are, respectively, $39 \%$ and $22 \%$, and the $\mathrm{SPA}_{C} p$-values are, respectively, $30 \%$, and $21 \%$. Hence, these returns are not statistically significant at any reasonable level. Note that the SPA $p$-values are all smaller than the Reality Check $p$ values, showing that the SPA tests are indeed more powerful. Nonetheless, the SPA tests

[^1]Table 2: Returns and Reality Check $p$-values of the rules with the largest mean return.

| Index | Rule | Daily | Annual | RC |
| :--- | :--- | :---: | :---: | :---: |
| DJIA | $\mathrm{MSV}^{a}$ | $0.058 \%$ | $14.67 \%$ | 0.39 |
| S\&P 500 | Contrarian OBV $^{b}$ | $0.061 \%$ | $15.38 \%$ | 0.22 |
| NASDAQ | 2-day MA |  |  |  |
| Russell 2000 | 2-day MA $^{d}$ | $0.152 \%$ | $38.19 \%$ | 0.00 |

Notes: The third and fourth columns are daily and annual returns; the last column is the Reality Check $p$-value.
$a$ : Momentum strategy in volume based on a moving average oscillator: 5-day moving average of 250-day ROC, $20 \%$ overbought/oversold rate, and 50 fixed holding days;
b: 10-5 day cross MA;
c: MA rule with multiplicative band 0.001;
$d$ : simple MA rule without multiplicative band.
are unable to reject the null hypothesis for these two indices. For NASDAQ Composite and Russell 2000, the best rules and best standardized rules are all short-term (2-day) MA rules. The annual returns of these best rules are much higher than those for DJIA and S\&P 500, and they are highly significant with $p$-values all close to zero. ${ }^{4}$

We also consider $1 \%$ significance level and use the associated Reality Check critical value to determine the number of rules with significant returns. ${ }^{5}$ As shown in Table 4, there exists a cluster of rules/strategies that are significantly profitable. This evidence is in line with the concept of "thick modeling" suggested by Granger and Jeon (2004) and Timmermann and Granger (2004). In addition, we observe the following. First, most of profitable rules and strategies for NSADAQ Composite and Russell 2000 are based on filter rules and moving averages rules. Second, no contrarian rule is significantly profitable. Third, there are much more profitable investor's strategies than simple rules ( 27 strategies vs. 6 simple rules for NASDAQ Composite and 161 vs. 35 for Russell

[^2]Table 3: Returns and SPA $p$-values of the rules with the largest standardized mean return.

| Index | Rule | Daily | Annual | $\mathrm{SPA}_{U}$ | $\mathrm{SPA}_{C}$ | $\mathrm{SPA}_{L}$ |
| :--- | :--- | :---: | :---: | :--- | :--- | :--- |
| DJIA | $\mathrm{MSV}^{a}$ | $0.058 \%(4.58)$ | $14.67 \%$ | 0.31 | 0.30 | 0.25 |
| S\&P 500 | $\mathrm{MSV}^{b}$ | $0.055 \%(4.83)$ | $13.98 \%$ | 0.21 | 0.21 | 0.18 |
| NASDAQ | 2-day MA |  |  |  |  |  |
| Russell 2000 | 2-day MA | $0.152 \%(8.02)$ | $38.19 \%$ | 0.00 | 0.00 | 0.00 |

Notes: The third and fourth columns are daily and annual returns with the numbers in parentheses being (standardized) test statistics; the last three columns are SPA $p$-values. $a$ : Momentum strategy in volume based on a moving average oscillator: 5-day moving average of 250 -day ROC, $20 \%$ overbought/oversold rate, and 50 fixed holding days; $b$ : The same as "a" above with $10 \%$ overbought/oversold rate;
$c$ and $d$ : MA rule with multiplicative band 0.001 .
2000). Note that these profitable strategies are all learning strategies. Fourth, and quite interestingly, there exist profitable strategies based on non-profitable simple rules. For example, no momentum strategy in volume is profitable for NASDAQ Composite, but there are 7 significantly profitable investor's strategies constructed from this class. For Russell 2000, significantly profitable strategies can also be constructed from the classes of support and resistance, channel break-outs and momentum strategies in price, even though there is no profitable simple rule in these classes. This shows that technical investors are able to make higher and significant profits by intelligently utilizing the information from simple rules. Therefore, the profitability of technical analysis should not be evaluated solely based on simple rules.

To see what rules and strategies make significant profit, we collect top 10 rules (ordered according to their mean returns) in Table 5; a complete table of all profitable rules and strategies in Table 4 is available upon request. It can be seen that there are also more investor's learning strategies ( 6 for NASDAQ Composite and 8 for Russell 2000). The second and third best rules for NASDAQ Composite are, respectively, a learning strategy based on the MSV class and a learning strategy based on the collection of all rules. These two returns are close to that of the best rule. For Russell 2000, there are six learning strategies based on the MA class. Except the best rule, the returns of the other nine rules are close to each other.

Table 4: Summary of significantly profitable rules and strategies in 1990-2000.

|  | NASDAQ Composite |  |  |  | Russell 2000 |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Class | Simple | Contrarian | Investor's |  | Simple | Contrarian | Investor's |
|  | Rules | Rules | Strategies |  | Rules | Rules | Strategies |
| FR | 2 | 0 | 2 | 22 | 0 | 50 |  |
| MA | 2 | 0 | 14 |  | 13 | 0 | 50 |
| SR | 0 | 0 | 0 |  | 0 | 0 | 7 |
| CB | 0 | 0 | 0 |  | 0 | 0 | 5 |
| OBV | 2 | 0 | 2 | 0 | 0 | 0 |  |
| MSP | 0 | 0 | 0 | 0 | 0 | 7 |  |
| MSV | 0 | 0 | 7 | 0 | 0 | 0 |  |
| LS-all | N/A | N/A | 2 | N/A | N/A | 42 |  |
| Total | 6 | 0 | 27 | 35 | 0 | 161 |  |

Notes: We list only the classes of significantly profitable rules and investor's strategies. LS-all is the class of learning strategies based on the collection of all 12 classes; the investor's strategies in other classes are all learning strategies. All rules and strategies are significant at $1 \%$ level based on the Reality Check critical value.

### 4.4 Comparison with the Buy-and-Hold Strategy

To confirm the profitability of technical analysis, we also compare the returns of the best rules identified in the preceding subsection (based on the in-sample data from 1990 through 2000) with that of the buy-and-hold strategy. Many studies had carried out such comparison, e.g., Fama and Blume (1966), Jensen and Benington (1970), Sweeney (1986, 1988) , and Levich and Thomas (1993). It is quite surprising to note that such comparisons were usually made without taking transaction costs into account. As we find that the best rules for NASDAQ Composite and Russell 2000 are short-term rules, the transaction costs resulted from frequent trading should not be overlooked. Without transaction costs, the profits of technical trading rules may not be reliable, as discussed in Bessembinder and Chan (1998). We thus consider transaction costs in evaluating the returns of the best rules.

The exact transaction costs of large institutional investors are difficult to measure after the deregulation in 1970s. Fama and Blume (1966) used the floor trader cost as the minimal transaction cost, which is estimated as $0.05 \%$ for each one-way trade. Whether

Table 5: Top 10 rules and strategies with significant returns.

| NASDAQ Composite |  |  | Russell 2000 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Rules <br> (Strategies) | Daily <br> Returns | Annual <br> Returns | Rules <br> (Strategies) | Daily <br> Returns | Annual <br> Returns |
| simple-MA | 0.1516\% | 38.19\% | simple-MA | 0.1864\% | 46.99\% |
| LS-MSV | 0.1510\% | $38.05 \%$ | LS-MA | 0.1775\% | $44.73 \%$ |
| LS-all | 0.1508\% | 38.01\% | LS-MA | 0.1775\% | $44.73 \%$ |
| LS-MSV | 0.1476\% | $37.21 \%$ | simple-MA | 0.1756\% | $44.24 \%$ |
| LS-OBV | 0.1458\% | $36.75 \%$ | LS-MA | 0.1749\% | 44.08\% |
| LS-MA | 0.1453\% | $36.62 \%$ | LS-MA | 0.1749\% | 44.08\% |
| LS-MA | 0.1453\% | 36.62\% | LS-FR | 0.1749\% | 44.08\% |
| simple-MA | $0.1448 \%$ | 36.49\% | LS-FR | 0.1749\% | 44.08\% |
| simple-OBV | $0.1448 \%$ | 36.49\% | LS-MA | 0.1736\% | 44.08\% |
| simple-OBV | 0.1435\% | $36.17 \%$ | LS-MA | 0.1736\% | $43.74 \%$ |

Notes: We list only the class titles of the rules and strategies but not their parameter values for a clear illustration. For example, simple-MA stands for a simple MA rule; LS-MA stands for a learning strategy based on the MA class; LS-all is a learning strategy based on the collection of all 12 classes of simple rules.
this cost rate is completely appropriate is still debatable. While Sweeney (1988) argues that this rate is overstated for the market after 1976, the other studies indicate the opposite. For example, Chan and Lakonishok (1993) estimate the commission cost for institutional traders in the largest decile of NYSE to be $0.13 \%$. Knez and Ready (1996) also obtain similar estimates for the average bid-ask spread actually paid in one-way trades for Dow Jones securities. In this paper, we follow Fama and Blume (1966) and deduct $0.05 \%$ from transaction price for each one-way trade. Such a cost rate is applicable for market-makers and may also be possible for large institutional investors. ${ }^{6}$

We summarize the comparison results in Table 6. When there is no transaction cost, it can be seen that the best rule for NASDAQ Composite outperforms the buy-andhold strategy in all in-sample periods except 1998 and 1999 (the best rule nevertheless generates positive profit in these two years). The best rule for Russell 2000 also results in

[^3]higher returns in all 11 years. When transaction costs are taken into account, the best rule for NASDAQ Composite has superior performance only in 7 out of 11 in-sample periods, yet the best rule for Russell 2000 still dominates in all in-sample periods. In terms of the average return over 11 years, the best rules for both indices beat the buy-and-hold strategy, regardless of the presence of transaction costs. For the out-of-sample periods 2001 and 2002, we still compare the performance of the best rules identified in the insample periods with that of the buy-and-hold strategy. Observe that the buy-and-hold strategy yields large negative annual returns for both indices, except for Russell 2000 in 2001 where the annual return ( $1 \%$ ) is barely positive. The best rule for NASDAQ Composite outperforms the buy-and-hold strategy, with positive returns in 2001 and smaller negative returns in 2002. The best rule for Russell 2000, on the other hand, generates larger positive returns in 2001 but larger negative returns in 2002. Although the best rules do not uniformly dominate the buy-and-hold strategy in all periods, it is fair to say that the best rules developed in-sample compare favorably with the buy-and-hold strategy in both in- and out-of-sample periods.

## 5 Conclusions

In this study, White's Reality Check and Hansen's SPA test are applied to re-examine the profitability of technical analysis based on a more complete set of trading rules and strategies. We find that significantly profitable simple rules and investor's strategies are available for the samples from relatively "young" markets (NASDAQ Composite and Russell 2000) but not for those of more "mature" markets (DJIA and S\&P 500). The latter finding supports the conclusion of STW which was based on Reality Check only. Compared with the buy-and-hold strategy, it is also found that such profitable rules and strategies can generate higher returns even when transaction costs are taken into account. Note that Hansen, Lunde, and Nason (2004) also found significant calendar effect in small stock indices. These results are consistent with Siegel (2002, pp. 290-297) and suggest that weak market efficiency has not yet formed in these "young" markets. This phenomenon may be due to the fact that the speed of information penetration and market liquidity are different in these two types of market. A careful study of why and to what extent market maturity affects market efficiency is an interesting future research direction.

Our empirical results also indicate the importance of investor's strategies in technical

Table 6: Annual returns of the best rules and the buy-and-hold strategy.

|  | NASDAQ Composite |  |  | Russell 2000 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Best Rule | Best Rule | Buy-and- |  | Best Rule | Best Rule | Buy-and- |
|  | w/o TC | with TC | Hold |  | w/o TC | with TC | Hold |
| 1990 | $67.6 \%$ | $58.6 \%$ | $-17.8 \%$ |  | $53.7 \%$ | $44.3 \%$ | $-21.5 \%$ |
| 1991 | $66.3 \%$ | $56.6 \%$ | $56.9 \%$ |  | $68.0 \%$ | $58.6 \%$ | $43.7 \%$ |
| 1992 | $35.1 \%$ | $25.0 \%$ | $15.5 \%$ |  | $37.7 \%$ | $27.7 \%$ | $16.4 \%$ |
| 1993 | $40.6 \%$ | $30.8 \%$ | $14.8 \%$ |  | $47.9 \%$ | $39.2 \%$ | $17.0 \%$ |
| 1994 | $33.2 \%$ | $23.7 \%$ | $-3.2 \%$ |  | $41.4 \%$ | $31.2 \%$ | $-3.2 \%$ |
| 1995 | $42.1 \%$ | $32.4 \%$ | $39.9 \%$ |  | $38.4 \%$ | $29.7 \%$ | $26.2 \%$ |
| 1996 | $40.9 \%$ | $30.3 \%$ | $22.7 \%$ |  | $30.1 \%$ | $19.7 \%$ | $14.8 \%$ |
| 1997 | $56.4 \%$ | $45.8 \%$ | $21.6 \%$ |  | $52.4 \%$ | $43.1 \%$ | $20.5 \%$ |
| 1998 | $27.1 \%$ | $15.9 \%$ | $39.6 \%$ |  | $82.7 \%$ | $72.9 \%$ | $-3.4 \%$ |
| 1999 | $8.9 \%$ | $-3.1 \%$ | $85.6 \%$ |  | $34.5 \%$ | $23.5 \%$ | $19.6 \%$ |
| 2000 | $3.1 \%$ | $-8.7 \%$ | $-39.3 \%$ |  | $31.5 \%$ | $19.8 \%$ | $-4.2 \%$ |
| $11-$ yr avg | $38.2 \%$ | $27.9 \%$ | $21.5 \%$ |  | $47.1 \%$ | $37.2 \%$ | $11.4 \%$ |
| 2001 | $28.9 \%$ | $19.3 \%$ | $-21.1 \%$ |  | $26.9 \%$ | $15.4 \%$ | $1.0 \%$ |
| 2002 | $-13.0 \%$ | $-22.4 \%$ | $-31.5 \%$ |  | $-39.8 \%$ | $-48.7 \%$ | $-21.9 \%$ |

Notes: w/o TC: without transaction costs; with TC: with transaction costs; 11-yr avg: the average annual return of 11 years of the in-sample period; 2001 and 2002 are out-of-sample periods.
analysis. We find that there are more significantly profitable investor's strategies than simple trading rules. More interestingly, we also find that technical investors are capable of constructing superior strategies from simple rules. In fact, Investor's strategies may generate significant profits from unprofitable simple rules. This shows that investor's learning and decision processes play a crucial role in technical analysis. Thus, investor's strategies should not be ignored in the studies of technical analysis. Merely rejecting the profitability of simple trading rules does not necessarily negate the usefulness of technical analysis.

## Appendix A: The Parameter Values of the Trading Rules and Strategies

## A. 1 Momentum Strategies in Price and in Volume

The parameters of the momentum strategies are:

$$
\begin{aligned}
& m(m \text {-day ROC })=2,5,10,20,30,40,50,60,125,250 \text { (10 values); } \\
& w(w \text {-day moving average })=2,5,10,20,30,40,50,60,125,250(10 \text { values }) ; \\
& k \text { (overbought/oversold level) }=0.05,0.10,0.15,0.2(4 \text { values }) ; \\
& f \text { (fixed holding days) }=5,10,25,50 \text { (4 values). }
\end{aligned}
$$

There are $10 m$ values and hence 10 simple oscillators. There are $10 w$ values. Setting $w$ less than or equal to $m$, we have 55 moving average oscillators. For cross-over moving average oscillators, there are 45 ratios of moving averages when $w_{1}<w_{2}$. We set $m=w_{2}$ and compute the moving averages of $w_{2}$-day ROC. Thus, there are 45 cross-over oscillators. We also set the overbought/oversold levels $k$ no higher than that recommended by Pring (1993, Chap. 3). We consider 4 fixed holding days $(f)$, as in STW. The total number of rules in the MSP class is thus $(10+55+45) \times 4 \times 4=1,760$. Similarly, there are 1,760 rules in the MSV class.

## A. 2 Head-and-Shoulders

There are 6 parameters in the HS class. In addition to $n, x$ and $f$ that are clear from the text, there are 3 more parameters: $k, r$ and $d$. An investor will not initiate a position until the closing price in following days falls below the right trough by a multiplicative constant $k$, known as multiplicative band. An investor will liquidate the short position when the the closing price in following days exceeds the right trough by a multiplicative constant $r$, known as the stoploss rate. The fixed liquidation price is the closing price that declines by an amount equal to $d$ times the head-trough difference, where the headtrough difference is calculated as the difference between the head and the average of two troughs. The parameters of the HS class are:

```
n(days of each subperiod) = 5, 10, 20, 50 (4 values);
x (differential rate of shoulders or troughs) = 0.005, 0.01, 0.015, 0.03, 0.05 (5
values);
```

```
k(multiplicative band) = 0, 0.005, 0.01, 0.02, 0.03 (5 values);
f(fixed holding days) = 5, 10, 25, 50 (4 values);
r(stoploss rate) = 0.005, 0.0075, 0.01, 0.015 (4 values);
d (parameter for fixed liquidation price) = 0.25, 0.5, 0.75,1 (4 values).
```

Given 4 values of $n, 5$ values of $x$, and 5 values of $k$, there are 100 combinations of $(n, x, k)$, and for each combination, there are 12 liquidation methods. The total number of rules in the HS class is thus $100 \times 12=1,200$.

Note: Lo, Mamaysky, and Wang (2000) recommended the differential rate $x=0.015$; Edwards and Magee (1997, p. 81) recommended the multiplicative band $k=0.03$; Chang and Osler (1999) recommended the stoploss rate $r=0.005,0.01$ and the fixed liquidation price parameter $d=0.25$. These parameter values are all included in our setup.

## A. 3 Triangle

There are 5 parameters in the TA class. The parameters $n, x, f$ and $r$ are as in the HS class. The trend filter $x=0$ means a position will be initiated upon completing a triangle. The investor will liquidate his/her position after the buy or sell signal lasts for $d$ days; the values of $d$ are set as STW. The parameters of the TA class are:

```
n(days of each subperiod) = 5,10, 20,50(4 values);
x ( \text { trend filter ) = 0, 0.001, 0.003, 0.005, 0.0075, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06,}
0.07, 0.08, 0.09, 0.1 (15 values);
f(fixed holding days) = 5, 10, 25, 50 (4 values);
r(stoploss rate) = 0.005, 0.0075, 0.01, 0.015 (4 values);
d (day filter) = 2, 3,4,5 (4 values).
```

Given 4 values of $n$ and 15 values of $x$, there are 60 combinations of $(n, x)$, and for each combination, there are 12 liquidation methods. The total number of rules in the TA class is $60 \times 12=720$.

## A. 4 Rectangle

There are 6 parameters in the RA class. The parameters $n, x, f, r$ and $d$ are as in the TA class. The parameter that determines whether the tops (bottoms) are near a horizontal line is $k$ (so that the bounds are $\pm k$ ). The parameters of the RA class are:

```
n(days of each subperiod) = 5, 10, 20, 50 (4 values);
k(parameter of bounds) = 0.005, 0.0075, 0.01 (3 values);
x ( \text { trend filter) = 0, 0.001, 0.003, 0.005, 0.0075, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06,}
0.07, 0.08, 0.09, 0.1 (15 values);
f(fixed holding days) = 5, 10, 25,50(4 values);
r(stoploss rate) = 0.005, 0.0075, 0.01, 0.015 (4 values);
d (day filter) = 2, 3, 4, 5 (4 values).
```

Given 4 values of $n, 3$ values of $k$, and 15 values of $x$, there are 180 combinations of ( $n, k, x$ ), and for each combination, there are 12 liquidation methods. The total number of rules in the RA class is $180 \times 12=2,160$.

## A. 5 Double Tops and Bottoms

There are 7 parameters in the DTB class. The parameters $x, f, r$ and $d$ are similar to those in the RA class. Following Lo, Mamaysky, and Wang (2000), each subperiod ( $n$ ) is at least 20-day (about 1 month). The parameter of bounds $k$ determines whether the tops (bottoms) are approximately equal; that is, each top (bottom) does not differ from the average of two tops (bottoms) for more than $\pm k$. For the gap rate $g$, the minimal (maximal) closing price of the second subperiod is below (above) the average of two tops (bottoms) with range $g$. Note that Edwards and Magee (1997) recommended the gap rate $g$ to be 0.15-0.2 (pp. 159-160) and the trend filter $x$ to be 0.03 (p. 161). The parameters of the DTB class are:

$$
\begin{aligned}
& n \text { (days of each subperiod })=20,40,60(3 \text { values }) ; \\
& k \text { (parameter of bounds })=0.005,0.01,0.015,0.03,0.05(5 \text { values }) ; \\
& g \text { (gap rate })=0.1-0.15,0.15-0.2,0.2-0.25(3 \text { values }) ;
\end{aligned}
$$

```
x(trend filter) = 0, 0.01, 0.02, 0.03 (4 values);
f(fixed holding days) = 5, 10, 25, 50 (4 values);
r(stoploss rate) = 0.005, 0.0075, 0.01, 0.015 (4 values);
d (day filter) = 2, 3, 4, 5 (4 values).
```

There are 180 combinations of $(n, k, g, x)$ and 12 liquidation methods. Thus, the total number of rules in the DTB class is $180 \times 12=2,160$.

## A. 6 Learning Strategies

There are 13 rule classes ( 12 classes of simple rules and the collection of all rules) and 3 performance measures. The other parameters of the LS class are:

$$
\begin{aligned}
& m(\text { memory span })=2,5,10,20,40,60,125,250 \text { days }(8 \text { values }) \\
& r(\text { review span })=1,5,10,20,40,60,125,250 \text { days }(8 \text { values })
\end{aligned}
$$

As $r \leq m$, there are 36 combinations of $(m, r)$. The total number of strategies in this class is $13 \times 3 \times 36=1,404$.

## A. 7 Vote Strategies

The parameters of the VS class are:

$$
\begin{aligned}
& m(\text { memory span })=1,2,5,10,20,40,60,125,250 \text { days }(9 \text { values }) ; \\
& r(\text { review span })=1,5,10,20,40,60,125,250 \text { days }(8 \text { values). }
\end{aligned}
$$

Given that $r \leq m$, there are 37 combinations of $(m, r)$. With 12 rule classes and 2 types of ballots, the total number of strategies in this class is $12 \times 2 \times 37=888$.

## References

Allen, F., and R. Karjalainen (1999). "Using Genetic Algorithms to Find Technical Trading Rules." Journal of Financial Economics 51, 245-271.

Bessembinder, H., and K. Chan (1998). "Market Efficiency and the Returns to Technical Analysis." Financial Management 27, 5-17.

Blume, L., D. Easley, and M. O'Hara (1994). "Market Statistics and Technical Analysis: The Role of Volume." Journal of Finance 49, 153-181.

Brock, W., J. Lakonishok, and B. LeBaron (1992). "Simple Technical Trading Rules and the Stochastic Properties of Stock Returns." Journal of Finance 47, 1731-1764.

Brown, S. J., W. N. Goetzmann, and A. Kumar (1998). "The Dow Theory: William Peter Hamilton's Track Record Reconsidered." Journal of Finance 53, 1311-1333.

Chan, K., A. Hameed, and W. Tong (2000). "Profitability of Momentum Strategies in the International Equity Markets." Journal of Financial and Quantitative Analysis 35, 153-172.

Chan, L. K. C., N. Jegadeesh, and J. Lakonishok (1996). "Momentum Strategies." Journal of Finance 51, 1681-1713.

Chan, L. K. C., N. Jegadeesh, and J. Lakonishok (1999). "The Profitability of Momentum Strategies." Financial Analysts Journal 55, 80-90.

Chan, L. K. C., J. Karceski, and J. Lakonishok (1998). "The Risk and Return from Factors." Journal of Financial and Quantitative Analysis 33, 159-188.

Chan, L. K. C., and J. Lakonishok (1993). "Institutional Trades and Intraday Stock Price Behavior." Journal of Financial Economics 33, 173-199.

Chang, P. H. K., and C. L. Osler (1999). "Methodological Madness: Technical Analysis and the Irrationality of Exchange-Rate Forecasts." Economic Journal 109, 636-661.
Edwards, R., and J. Magee (1997). Technical Analysis of Stock Trends, 7th ed., New York, NY: John Magee Inc.
Fama, E. F., and M. E. Blume (1966). "Filter Rules and Stock-Market Trading." Journal of Business 39, 226-241.

Gencay, R. (1998). "The Predictability of Security Returns with Simple Technical Trading Rules." Journal of Empirical Finance 5, 347-359.

Gencay, R. (1999). "Linear, Non-Linear and Essential Foreign Exchange Rate Prediction
with Simple Technical Trading Rules." Journal of International Economics 47, 91107.

Goetzmann, W. N., and M. Massa (2002). "Daily Momentum and Contrarian Behavior of Index Fund Investors." Journal of Financial and Quantitative Analysis 37, 375-390.

Granger, C. W. J., and Y. Jeon (2004). "Thick Modeling," Economic Modeling 21, 323-343.

Hansen, P. R. (2004). "A Test for Superior Predictive Ability." Working paper, Department of Economics, Stanford University.

Hansen, P. R., A. Lunde, and J. M. Nason (2004). "Testing the Significance of Calendar Effects," Working paper, Department of Economics, Stanford University.

Jensen, M. C., and G. A. Benington (1970). "Random Walks and Technical Theories: Some Additional Evidence." Journal of Finance 25, 469-482.

Knez, P., and M. Ready (1996). "Estimating the Profits from Trading Strategies." Review of Financial Studies 9, 1121-1164.

Lakonishok, J., A. Shleifer, and R. W. Vishny (1994). "Contrarian Investment, Extrapolation, and Risk." Journal of Finance 49, 1541-1578.

Lakonishok, J., and S. Smidt (1988). "Are Seasonal Anomalies Real? A Ninety-Year Perspective." Review of Financial Studies 1, 403-425.

LeBaron, B. (1992). "Persistence of the Dow Jones Index on Rising Volume." Working Paper, Department of Economics, University of Wisconsin, Madison.

LeBaron, D., and R. Vaitilingam (1999). The Ultimate Investor. Dover, NH: Capstone Publishing.

Levich, R. M., and L. R. Thomas (1993). "The Significance of Technical Trading-Rule Profits in the Foreign Exchange Market: A Bootstrap Approach." Journal of International Money and Finance 12, 451-474.

Lo, A. W., and A. C. MacKinlay (1990). "Data-Snooping Biases in Tests of Financial Asset Pricing Models." Review of Financial Studies 3, 431-467.

Lo, A. W., H. Mamaysky, and J. Wang (2000). "Foundations of Technical Analysis: Computational Algorithms, Statistical Inference, and Empirical Implementation." Journal of Finance 55, 1705-1765.

Merton, R. C. (1987). "On the Current State of the Stock Market Rationality Hypothesis." in R. Dornbusch, S. Fischer, and J. Bossons (eds.), Macroeconomics and

Finance: Essays in Honor of Franco Modigliani. Cambridge, MA: MIT Press.
Neely, C. J., and P. Weller (1999). "Technical Trading Rules in the European Monetary System." Journal of International Money and Finance 18, 429-458.

Neely, C. J., P. Weller, and R. Dittmar (1997). "Is Technical Analysis in the Foreign Exchange Market Profitable? A Genetic Programming Approach." Journal of Financial and Quantitative Analysis 32, 405-426.

Politis, D. N., and J. P. Romano (1994). "The Stationary Bootstrap." Journal of the American Statistical Association 89, 1303-1313.

Pring, M. J. (1991). Technical Analysis Explained, Third ed., New York, NY: McGrawHill.

Pring, M. J. (1993). Martin Pring on Market Momentum. New York, NY: McGraw-Hill.
Ready, M. J. (2002). "Profits from Technical Trading Rules." Financial Management 31, 43-61.

Rouwenhorst, K. G. (1998). "International Momentum Strategies." Journal of Finance 53, 267-284.

Rouwenhorst, K. G. (1999). "Local Return Factors and Turnover in Emerging Stock Markets." Journal of Finance 54, 1439-1464.

Siegel, J. J. (2002). Stocks for the Long Run, Third Ed., New York, NY: McGraw-Hill.
Sullivan, R., A. Timmermann, and H. White (1999). "Data-Snooping, Technical Trading Rule Performance, and the Bootstrap." Journal of Finance 54, 1647-1691.

Sullivan, R., A. Timmermann, and H. White (2001). "Dangers of Data-Driven Inference: The Case of Calendar Effects in Stock Returns." Journal of Econometrics 105, 249-286.

Sweeney, R. J. (1986). "Beating the Foreign Exchange Market." Journal of Finance 41, 163-182.

Sweeney, R. J. (1988). "Some New Filter Rule Tests: Methods and Results." Journal of Financial and Quantitative Analysis 23, 285-300.

Timmermann, A., and C. W. J. Granger (2004). "Efficient Market Theory and Forecasting." International Journal of Forecasting 20, 15-27.

White, H. (2000). "A Reality Check for Data Snooping." Econometrica 68, 1097-1126.


[^0]:    ${ }^{1}$ This is known as "double-or-out" (Bessembinder and Chan, 1998); we are indebted to A. Timmermann for helpful suggestions on this issue.
    ${ }^{2}$ We adopt "wrap-up" resampling such that the first row $\left(\eta_{1,1}, \ldots, \eta_{1, M}\right)$ is treated as the next row of the last row $\left(\eta_{n, 1}, \ldots, \eta_{n, M}\right)$ in resampling.

[^1]:    ${ }^{3}$ We thank P. R. Hansen for useful discussion on the choice of the variance estimator.

[^2]:    ${ }^{4}$ In the early version of this paper, we also examined the Sharpe ratio of these indices by Reality Check. Although the rules with the largest Sharpe ratio may be different from the best rules here, the same conclusion holds. That is, there are signficant Sharpe ratios for NASDAQ Composite and Ruseell 2000 but not for the other two indices.
    ${ }^{5}$ As the best rules for NASDAQ Composite and Russell 2000 all have RC and SPA p-values close to zero, the RC and SPA distributions appear to be quite close at extreme right tails. Thus, the RC critical values with a small significance level (1\%) should provide reasonable cutoff values for identifying significant rules.

[^3]:    ${ }^{6}$ Nevertheless, it is understood that this cost is underestimated for non-floor traders because other costs, such as brokerage commissions and bid-ask spreads, are inevitable. We are indebted to C. Jones and B. Lehmann for helpful discussions on this point.

